

Signatures of d-Wave Symmetry on Thermal Dirac Fermions in Graphene-Based F/I/d Junctions

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(Dated: 2 November 2011)

We study theoretically the behavior of thermal massless Dirac fermions inside graphene-based Ferromagnetic | Insulator | d -wave/ s -wave superconductor (F|I| d and F|I| S) junctions in the ballistic regime. Using the Dirac-BdG wave functions within the three regions and appropriate boundary conditions, the Andreev and Normal reflection coefficients are derived. By employing the obtained Andreev and Normal reflection coefficients the characteristics of heat current through the F|I| d and F|I| S junctions are investigated within the thin barrier approximation. We find that for s -wave superconductors, thermal conductance oscillates sinusoidally vs barrier strength. The finding persist for the values of α , the orientation of d -wave superconductor crystal in the k -space, below $\pi/4$. By increasing temperature, the thermal conductance is increased exponentially for small values of α and for larger values the quantity is modified to exhibit a linear behavior at $\alpha = \pi/4$ which is similar to Wiedemann-Franz law for metals in low temperatures.

I. INTRODUCTION

Graphene is a single layer of carbon atoms which was introduced to the scientific community by Novoselov *et al.* in 2004^{1,2}. Most of applicable and interesting characteristics of graphene has been investigated intensely experimentally¹⁻⁶. Because of interesting phenomena which graphene showed, the artificial material has been received robust attentions from theoretical and experimental physics communities⁷⁻¹⁰. Induction of superconductive correlations into graphene layer by proximity of superconducting electrodes observed experimentally by Heersche *et al.*¹¹. Also inducing the ferromagnetism into graphene layers by means of proximity effects observed experimentally by Tombros *et al.*¹². In this regard, theoretical scientists utilized Dirac-Bogoliubov-de Gennes (Dirac-BdG) for investigating and predicting interesting phenomena because of the proximity effects^{7,9,13-16}.

Interplay between ferromagnetic graphene sheets and conventional superconductor is generalized theoretically¹⁵. Also Linder *et al.*¹⁷ generalized the theoretical investigations from conventional to unconventional superconductors in similarity with metallic cases which has been studied intensely¹⁸⁻²⁰. Most of the previous works are devoted to study electronic transport properties of the graphene-based junctions such as; Josephson currents^{8,21}, electronic conductance^{9,24}, spintronic conductance^{22,23}, shot noise^{10,26} and etc., but poor attentions has been focused on the heat transport properties and electronic thermal conductance of the junctions. The BTK formalism is generalized by Bardas and Averin²⁷ for obtaining electronic thermal conductance in the clean limit. Also, Devyatov

et al.^{28,29} studied electronic thermal conductance of Normal metal|I| d junctions in the ballistic regime. For high-sensitive devices including graphene-based junctions, knowing all electronic and thermal properties of the junctions are crucial and important points from application point of view. Few previous works devoted for investigating of electronic thermal transport characteristics of the graphene-based junctions^{30,31}.

In this paper we especially investigate signatures of $d_{x^2-y^2}$ -wave symmetry on the electronic heat transport characteristics of the F|I| d junctions. We start with the Dirac-BdG Hamiltonian and use obtained wave functions within the three regions and appropriate boundary conditions at interfaces for deriving the Andreev and Normal reflection coefficients in the thin barrier approximation^{24,25}. Using the mentioned coefficients we present numerical investigations of electronic thermal conductance of the Ferromagnetic|Insulator| d / s -wave superconductor junctions in the ballistic limit. We find that for F|I| d junctions the electronic thermal conductance, Γ shows oscillatory behavior vs barrier strength. Increasing the orientation angle of the superconducting gap α up to values close to $\pi/4$ only enhances whole values of the Γ and for maximum value of the superconducting gap orientation angle namely $\alpha = \pi/4$ magnitude of the oscillations diminish highly. Another finding is that the electronic thermal conductance shows an exponential increase vs temperature for small values of α . By increasing the angle of superconducting gap orientation up to $\pi/4$ the mentioned exponential form modify to linear form and at $\alpha = \pi/4$ the thermal conductance shows precisely linear increase with respect to temperature, namely $\Gamma \propto T$ that the finding induces in mind the Wiedemann-Franz law³² from metals in the low temperatures. The paper is organized as follows:

In Sec. II we explain the analytical derivations of Andreev and Normal reflection coefficients by starting from the Dirac-BdG Hamiltonian and in Sec. III the electronic thermal conductance of F|I| S is investigated by plotting

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the quantity with respect to strengthes of exchange field and barrier in the thin barrier approximation regime. In Sec. IV we study the effects of $d_{x^2-y^2}$ -wave symmetry on the electronic thermal conductance of the junctions within the thin barrier approximation. The paper go to end with conclusions in Sec. V.

II. THEORY

We study interplay between graphene-based ferromagnetic and superconductor junctions in the ballistic limit, therefore we employ the Dirac-BdG Hamiltonian for obtaining suitable wave functions. The general Dirac-BdG equation incorporating ferromagnetism and superconductivity reads as¹⁵:

$$\begin{pmatrix} H_0 - \sigma h & \Delta(T) \\ \Delta^*(T) & -(H_0 - \bar{\sigma} h) \end{pmatrix} \begin{pmatrix} u_\sigma \\ v_{\bar{\sigma}} \end{pmatrix} = \epsilon_\sigma \begin{pmatrix} u_\sigma \\ v_{\bar{\sigma}} \end{pmatrix},$$

$$H_0(\mathbf{r}) = -i\hbar v_F(\sigma_x \partial_x + \sigma_y \partial_y) + U(\mathbf{r}) - E_F \quad (1)$$

where σ_x and σ_y are 2×2 Pauli matrices and $\Delta(T)$ stands for temperature-dependent order parameter of superconducting region, also h represents the strength of exchange field in the ferromagnetic region. ϵ_σ stands for excitation energies of holelike and electronlike quasiparticles. For obtaining Dirac-BdG wave functions in the Normal, ferromagnetic and superconducting region, one should set $\{h = 0, \Delta(T) = 0\}$, $\{h \neq 0, \Delta(T) = 0\}$ and $\{h = 0, \Delta \neq 0\}$ inside Eq. (1), respectively. Here $\sigma = \pm 1$ stands for spin-up and -down quasiparticles and $\bar{\sigma} = -\sigma$. Also $U(\mathbf{r})$ shows the Fermi energy mismatch. Throughout the paper we consider a step function for spatial-dependency of the superconducting gap, namely $\Delta(x, T) = \Delta(T)\Theta(x)$ in which $\Theta(x)$ is the well known step function. By solving the Eq. (1) in the Ferromagnetic region we obtain Dirac-BdG wave functions for electronlike and holelike quasiparticles as follows:

$$\begin{cases} \psi_{e,\sigma}^\pm(x) = \frac{1}{\sqrt{\cos\theta_\sigma}} (1, \pm e^{\pm i\theta_\sigma}, 0, 0)^T e^{\pm i k_{e,\sigma} x} \\ \psi_{h,\bar{\sigma}}^\pm(x) = \frac{1}{\sqrt{\cos\theta_{\bar{\sigma}}}} (0, 0, 1, \mp e^{\pm i\theta'_{\bar{\sigma}}})^T e^{\pm i k_{h,\bar{\sigma}} x}, \end{cases} \quad (2)$$

where $\theta_\sigma(\theta'_{\bar{\sigma}})$ are propagation angles of electronlike (holelike) quasiparticles with respect to the normal trajectory into the interface at $x = 0$. We define the two incident angles as

$$\begin{cases} \theta_\sigma = \arcsin\left(\frac{\hbar v_F q}{\epsilon + E_F + \sigma h}\right) \\ \theta'_{\bar{\sigma}} = \arcsin\left(\frac{\hbar v_F q}{\epsilon - E_F + \sigma h}\right), \end{cases} \quad (3)$$

and x -components of the wave vectors for electronlike and holelike quasiparticles in the Ferromagnetic region are obtain as

$$\begin{cases} k_{e,\sigma} = \frac{\epsilon + E_F + \sigma h}{\hbar v_F} \cos\theta_\sigma \\ k_{h,\bar{\sigma}} = \frac{\epsilon - E_F + \sigma h}{\hbar v_F} \cos\theta'_{\bar{\sigma}}, \end{cases} \quad (4)$$

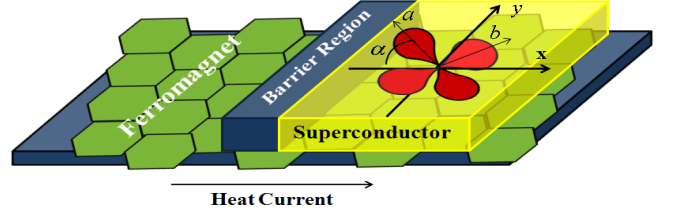


FIG. 1. (Color online) Model for suggested set up of Ferromagnetic|Barrier| d -wave superconductor graphene-based junctions. The junction is located at $x = 0$. Axes of d -wave superconductor crystal are represented with a and b . The superconductor crystal orientation angle with respect to normal trajectory (x -axis) is shown by α .

in which E_F and q are Fermi energy and y -component of wave vector in the Ferromagnetic region, respectively. For normal region with a barrier potential V_0 , it is sufficient to set $h = 0$ and $E_F \rightarrow (E_F - V_0)$ in the above obtained equations inside the Ferromagnetic region. The barrier potential V_0 can be applied by a gate voltage into the region. Within the d -wave superconductor region ($x > 0$), Dirac-BdG wave functions for electronlike and holelike quasiparticles are obtained:

$$\begin{cases} \psi_{S,e}^+ = (e^{i\beta_+}, e^{i\beta_+ + i\gamma_+}, e^{-i\phi_+}, e^{i\gamma_+ - i\phi_+})^T e^{-i(k_0 - i\chi_+)x} \\ \psi_{S,h}^- = (e^{-i\beta_-}, -e^{-i\beta_- - \gamma_-}, e^{i\phi_-}, -e^{-i\gamma_- - i\phi_-})^T e^{-i(k_0 - i\chi_-)x} \end{cases} \quad (5)$$

where we define $\chi_\pm = (U_0 + E_F) \sin\beta_\pm / k_0 (\hbar v_F)^2$ in which k_0 is defined as $k_0 = ((\frac{U_0 + E_F}{\hbar v_F})^2 - q^2)^{1/2}$. In Eq. (5) $e^{i\beta_\pm}$ is defined as u_\pm / v_\pm and

$$\begin{cases} u_\pm = \sqrt{\frac{1}{2} \left(1 + \frac{\sqrt{\epsilon^2 - |\Delta(\gamma_\pm)|^2}}{\epsilon} \right)} \\ v_\pm = \sqrt{\frac{1}{2} \left(1 - \frac{\sqrt{\epsilon^2 - |\Delta(\gamma_\pm)|^2}}{\epsilon} \right)} \end{cases} \quad (6)$$

$$\beta_\pm = \begin{pmatrix} \cos^{-1}\left(\frac{\epsilon}{\Delta(\gamma_\pm)}\right), & \epsilon < \Delta(\gamma_\pm) \\ -i \cosh^{-1}\left(\frac{\epsilon}{\Delta(\gamma_\pm)}\right), & \epsilon > \Delta(\gamma_\pm) \end{pmatrix}, \quad (7)$$

$$e^{i\phi_\pm} = \frac{\Delta(\gamma_\pm)}{|\Delta(\gamma_\pm)|} \quad (8)$$

$$\begin{cases} \gamma_+ = \arcsin \frac{\hbar v_F q}{U_0 + E_F} \\ \gamma_- = \pi - \arcsin \frac{\hbar v_F q}{U_0 + E_F} \end{cases} \quad (9)$$

In the case of d -wave symmetry, the orientational dependency of superconducting gap reads as $\Delta(\gamma_\pm) = \Delta(T) \cos(2\gamma_\pm - 2\alpha)$ in which α represents the orientation angle of d -wave superconducting gap. We now proceed and using the above Dirac-BdG wave functions and appropriate boundary conditions derive the Andreev and

Normal reflection coefficients. By applying appropriate boundary conditions for the two interfaces which are located at $x = 0$ and L , we obtain all reflection and transmission coefficients. At last we assume that a large gate voltage $V_0 \gg 1$ is applied into the narrow ($L \ll 1$) normal region. In this case $\Omega = V_0 L / \hbar v_F$ is a constant which is called strength of barrier. The approximation is called thin barrier approximation regime in which the Normal region acts as an insulator. The Andreev and Normal reflection coefficients in the thin approximation regime are derived which are available in Appendix A for F|I|s-wave superconductor junctions. We assume a right-going electronlike quasiparticle within the ferromagnetic region incident into interface between the ferromagnetic and Insulator regions, so the appropriate boundary condition in the interfaces at $x = 0$ is:

$$\begin{aligned} \psi_{e,\sigma}^+(x) + r_{A,\bar{\sigma}} \psi_{h,\bar{\sigma}}^-(x) + r_{N,\sigma} \psi_{e,\sigma}^-(x) = \\ t_{I,e}^+ \psi_{I,e}^+(x) + t_{I,e}^- \psi_{I,e}^-(x) + t_{I,h}^+ \psi_{e,h}^+(x) + t_{I,h}^- \psi_{e,h}^-(x), \end{aligned} \quad (10)$$

and other boundary condition in interface between the insulator and superconductor regions at $x = L$ is:

$$\begin{aligned} t_{I,e}^+ \psi_{I,e}^+(x) + t_{I,e}^- \psi_{I,e}^-(x) + t_{I,h}^+ \psi_{e,h}^+(x) + t_{I,h}^- \psi_{e,h}^-(x) = \\ t_{S,e}^+ \psi_{S,e}^+(x) + t_{S,h}^- \psi_{S,h}^-(x) \end{aligned} \quad (11)$$

where $r_{A,\bar{\sigma}}$ and $r_{N,\sigma}$ are amplitudes of spin-dependent Andreev and Normal reflection coefficients within the ferromagnetic region, respectively. Other coefficients are transmission coefficients in the Normal and superconducting regions. By applying the thin barrier approximation on the obtained reflection and transmission factors they reduce to simple factors that are available in the Appendix A. For investigating the electronic thermal conductance of the junction one needs to calculate the probabilities of Andreev and Normal reflections namely $|r_{A,\bar{\sigma}}|^2$ and $|r_{N,\sigma}|^2$. By assuming a temperature gradient through the junction, the normalized thermal conductance $\Gamma = \Gamma' / \Gamma_0$ is given as follow^{14,27}:

$$\begin{aligned} \Gamma' / \Gamma_0 = \sum_{\sigma=\uparrow\downarrow} \int_0^\infty \int_{-\pi/2}^{\pi/2} d\epsilon d\theta_\sigma \cos(\theta_\sigma) \{ 1 - |r_{N,\sigma}(\epsilon, \theta_\sigma)|^2 \\ - |r_{A,\sigma}(\epsilon, \theta_\sigma)|^2 \} \frac{\epsilon^2}{T^2 \cosh^2(\frac{\epsilon}{2T})}, \end{aligned} \quad (12)$$

where $\Gamma_0^{-1} = 2\pi^2 \hbar^2 v_F k_B \Delta_0 / E_F$ is a constant. We proceed to investigate the characteristics of electronic heat transport Γ of the mentioned junctions and throughout the paper we normalize energies with respect to Δ_0 and we set $\Delta_0 = \hbar = k_B = 1$ throughout our computations.

III. ELECTRONIC THERMAL CONDUCTANCE OF THE F|I|S JUNCTIONS IN THE THIN BARRIER APPROXIMATION

In this section we study electronic thermal transport characteristics of the Ferromagnetic|Insulator|s-wave superconductor junctions in the ballistic and thin barrier

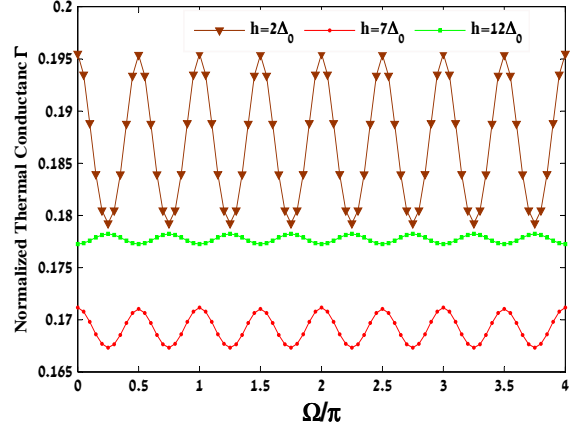


FIG. 2. (Color online) The normalized thermal conductance of F|I|S graphene-based junctions vs. normalized strength of barrier region Ω/π for three values of $h = 2\Delta_0, 7\Delta_0, 12\Delta_0$. The temperature has been fixed at $T = 0.2T_c$.

approximation regime. In Fig. 2 we set $T = 0.2T_c$, and plot normalized thermal conductance Γ vs normalized strength of barrier Ω/π for three different values of magnetization texture strength h/Δ_0 , also in Fig. 3 $\Omega = 0$ is set and the normalized conductance is plotted for three values of temperatures vs magnetization texture strength h/Δ_0 . Throughout our calculations we have set $E_F = 10\Delta_0$ and also used a large mismatch potential U_0 . The normalized thermal conductance shows an oscillatory behavior vs Ω/π which this finding can be understood by noting the fact that how the amplitude of Andreev and Normal reflections depend on Ω , (See Appendix A). In the thin barrier approximation, the width of normal layer L and barrier potential V_0 set for small and large values, respectively. Andreev and Normal coefficients are involved $\cos 2\Omega$ and $\sin 2\Omega$ terms which are periodic functions of Ω and consequently the appeared periodic oscillations in the thermal conductance are originated from the two appeared periodic functions in the Andreev and Normal coefficients. As it can be seen in Fig. 1, since the configuration contains one semi-infinite superconductor the Andreev bound states don't contribution to the transport characteristics of the junction under consideration^{7,8,24,25}. For small values of magnetic strength, the amplitude of oscillations has been enhanced in comparison with larger values of h/Δ_0 .

By increasing the magnetic strength, incident angle defined in Eq. (3) reduce and hence diminish the amplitude of oscillations which means suppression of available propagating channels in the system. The normalized thermal conductance of the F|I|S junction is plotted vs. h/Δ_0 , the magnetization strength of Ferromagnetic region, in Fig. 3 for three values of $T = 0.2T_c, 0.5T_c, 0.7T_c$ and also $\Omega = 0$ is set for the three plots. The thermal conductance shows a minimum at $h \simeq E_F$ and by increasing temperature move the minimum towards smaller values of h . The magnetization texture splits Fermi level into two parts in

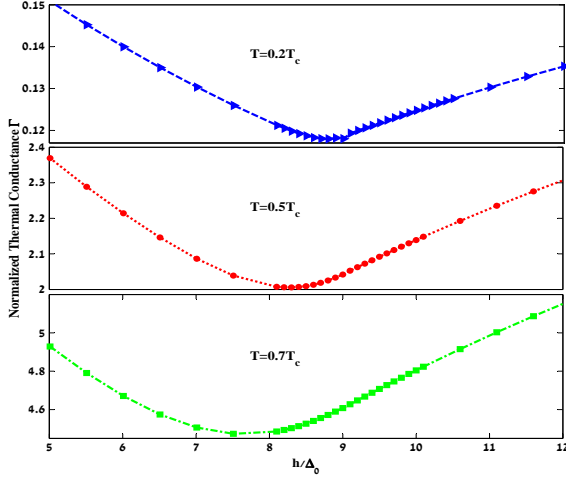


FIG. 3. (Color online) The normalized heat conductance of F|I|S graphene-based junctions vs magnetic exchange field strength h/Δ_0 of Ferromagnetic region for three values of temperatures, $T = 0.2T_c, 0.5T_c, 0.7T_c$ and fixed barrier strength at $\Omega = 0$.

the k -space and by increasing h , the two parts separate upward and downward more and more. Increasing the exchange splitting suppress propagating Dirac Fermions modes in the configuration under consideration up to values near $h \simeq E_F$, for larger values of h the propagating channels enhance, see Ref.s 14 and 15. In the light of above discussion, the thermal conductance reach to its minimum value at $h \simeq E_F$ that depends on the temperature. The fact also can be inferred from Fig. 2 which the curve of $h = 12\Delta_0$ has an intermediate value between the curves of $h = 2\Delta_0$ and $h = 7\Delta_0$. We proceed to investigate effects of d -wave symmetry on the heat conductance of F|I|d junctions in the clean limit.

IV. ELECTRONIC THERMAL CONDUCTANCE OF THE F|I|D JUNCTIONS IN THE THIN BARRIER APPROXIMATION

Now we present main results of this paper namely the fingerprints of $d_{x^2-y^2}$ -wave superconducting region on the electronic heat transport characteristics of F|I|d junctions in the thin barrier regime whose interfaces are located at $x = 0, L$. As it is seen in Fig. 1, unlike s -wave superconductors, the role of crystal orientation of $d_{x^2-y^2}$ -wave superconductors with respect to interface is very important. We assume a two-dimensional d -wave superconductor with cylindrical Fermi surface in the k -space is deposited on top of graphene sheet and connected to a sandwiched insulator region between Ferromagnetic and superconducting regions. The pair potential for s -wave superconductor is isotropic *i.e.* $\Delta(T) = \Delta_0 \tanh \sqrt{1.76 \sqrt{T_c/T} - 1}$. On the other hand, the pair potential for $d_{x^2-y^2}$ -wave symmetry is

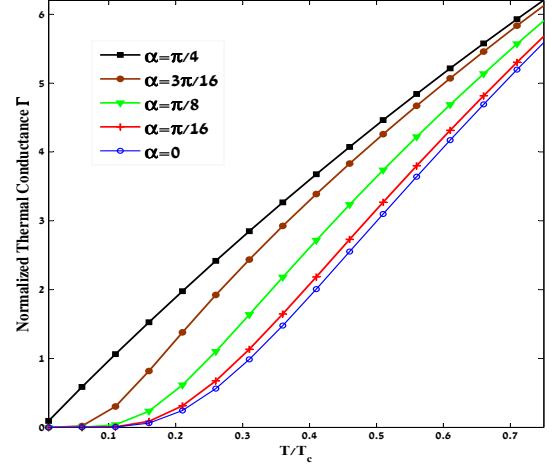


FIG. 4. (Color online) The normalized thermal conductance Γ of F|I|d graphene-based junctions vs temperature for five values of d -wave superconducting gap orientation $\alpha = 0, \pi/16, \pi/8, 3\pi/16, \pi/4$. The strength of magnetic exchange field and barrier fixed at $h = 2\Delta_0$, $\Omega = 0$, respectively.

θ -dependent, namely angle between the a -axis of the superconductor crystal and wavevector of the conducting quasiparticles. In this case, superconducting gap is anisotropic *i.e.* $\Delta_{\pm}(T, \gamma) = \Delta_d(T) \cos(2\gamma \pm 2\alpha)$ in which α is angle of a -axis with respect to normal trajectory to the interface (See Fig. 1) and γ is propagation angle of quasiparticles. The temperature dependency of d -wave superconductors is different from s -wave case³³. As it mentioned above, for the thin barrier approximation is assumed that $L \ll 1$ and $V_0 \gg 1$, so one can consider Ω as a constant and terms involving Ω reduce to simpler ones. Here we have set $E_F = 10\Delta_0$ and use large mismatch potential U_0 . Fig. 4 indicates electronic heat conductance of the F|I|d junctions vs temperature for five different values of crystal orientation of $d_{x^2-y^2}$ -wave superconductor α , the exchange field and strength of thin barrier are set at $h = 2\Delta_0$ and $\Omega = 0$, respectively. Electronic thermal conductance for $\alpha = 0, \pi/16$ shows an exponential increase vs temperature and for larger values of α , the exponential form is modified to linear increase. *The heat conductance shows completely linear increase vs temperature at maximum value of superconductor crystal orientation angle $\alpha = \pi/4$ that induces in mind the Wiedemann-Franz law for metals in low temperatures which thermal conductance is proportional to temperature, $\Gamma \propto T$.* The finding is arisen from orientational-dependent superconducting gap that increasing α decreases the propagating channels of superconducting correlations described by Andreev reflection coefficients. Although the Dirac and Schrodinger equations are used in graphene-based and metallic junctions respectively but the behaviors of thermal conductance in the graphene-based junctions are qualitatively similar to results of metallic N|I|d-wave junctions in which the propagating channels of moving quasi-particles is closed

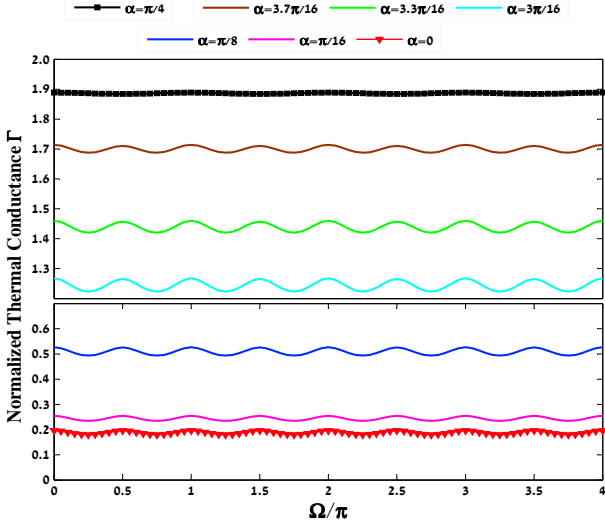


FIG. 5. (Color online) The normalized thermal conductance Γ of F|I|d graphene-based junctions vs normalized strength of barrier region Ω for seven values of superconducting gap orientation $\alpha = 0, \pi/16, \pi/8, 3\pi/16, 3.3\pi/16, 3.7\pi/16, \pi/4$. The strength of exchange field and temperature are fixed at $h = 2\Delta_0$, $T = 0.2T_c$, respectively.

by increasing crystal orientation angle from 0 to $\pi/4$ in Ref.s 28 and 29. In this context d -wave symmetry shows the same effects on thermal conductance of both graphene-based and metallic junctions. The behaviors of heat conductance vs strength of barrier are shown in Fig. 5 for several values of α . Temperature and exchange field have set in $h = 2\Delta_0$ and $T = 0.2T_c$. The thermal conductance vs strength of barrier region shows an oscillatory behavior and the increase of α enhance whole values of heat conductance. The period of oscillations vs strength of barrier suppresses completely for maximum crystal orientation angle $\alpha = \pi/4$. In Fig. 6 the thermal conductance is plotted vs the strength of magnetization exchange field h/Δ_0 for several values of α and $T = 0.2T_c$. In general, Γ for F|I|d junctions vs h/Δ_0 behaves similar to F|I|S configuration. The behavior can be verified by noting the mentioned reasons in the Sec. III for F|I|S case. Increasing the crystal orientation angle of d -wave superconductor up to $\alpha = \pi/4$ can only enhance whole values of the thermal conductance vs h/Δ_0 .

V. SUMMARY

In summary we have considered Ferromagnetic|Barrier| s/d -wave superconductors graphene-Based junctions in the thin barrier approximation and ballistic limit. We have utilized the Dirac-BdG equation and by employing Dirac-BdG wavefunctions derived the Andreev and Normal reflection amplitudes. Electronic thermal conductance Γ , of the two mentioned junctions in the thin barrier approximation has been

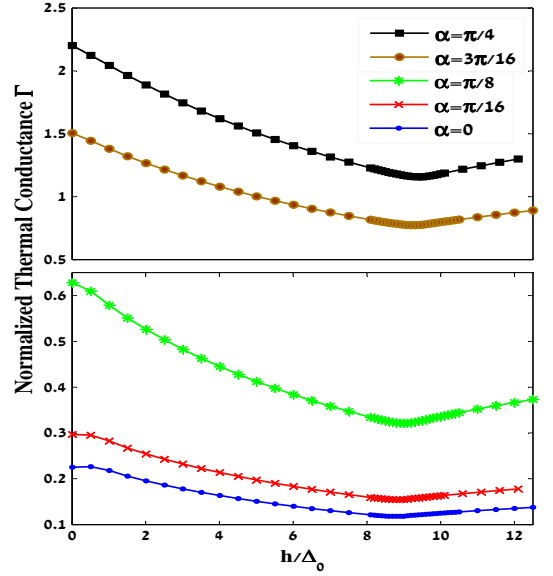


FIG. 6. (Color online) The normalized thermal conductance of F|I|d graphene-based junctions vs strength of magnetic exchange field h/Δ_0 for five values of superconducting gap orientation $\alpha = 0, \pi/16, \pi/8, 3\pi/16, \pi/4$. The temperature and strength of barrier are fixed at $T = 0.2T_c$, $\Omega = 0$, respectively.

investigated as well. We found that for F|I|S junctions, the heat conductance vs magnetization strength h/Δ_0 shows a minimum at values near $h \simeq E_F$ that by increasing temperature the minimum move towards smaller values of h/Δ_0 . The finding is qualitatively similar to F|I|d junctions but increasing superconductive gap orientation α shifts whole values of Γ towards larger values and no change induces to the trend of Γ vs h/Δ_0 . The electronic thermal conductance vs barrier strength oscillates and shows identical behavior for F|I|S and F|I|d configurations for all values of superconductor crystal orientation α except values near $\alpha \simeq \pi/4$. By approaching to $\alpha = \pi/4$, the propagating channels diminish and hence the amplitude of oscillations suppress. *We found a Wiedemann-Franz law-like in the low temperature regime for thermal conductance of F|I|S junctions, namely $\Gamma \propto T$.* The electronic heat conductance shows an exponential growth vs temperature for small values of gap orientation angle $\alpha < \pi/8$ and for larger values especially at $\alpha = \pi/4$ approaches to completely linear growth, namely $\Gamma \propto T$.

ACKNOWLEDGMENTS

We thankful very useful and fruitful discussions with Jacob Linder. The authors would like to thank the Office of Graduate Studies of Isfahan University.

Appendix A: Andreev and Normal reflection coefficients in the thin barrier approximation regime for F|I|S junctions

Using the boundary condition Eq.s (10, 11) and applying the thin barrier approximation, the Andreev and Normal reflection coefficients for the F|I|S junctions are obtained as follows;

$$r_A = \frac{\sqrt{\cos \theta_\sigma} \sqrt{\cos \theta'_\sigma} \cos \gamma e^{i(\frac{\theta_\sigma + \theta'_\sigma}{2})}}{\Upsilon_1 + i\Upsilon_2}$$

$$r_N = \frac{e^{i\theta_\sigma} (\Sigma_1 + i\Sigma_2)}{\Upsilon_1 + i\Upsilon_2}$$

$$\Sigma_1 = \cos 2\Omega \cos\left(\frac{\theta_\sigma + \theta'_\sigma}{2}\right) \sin \beta \sin \gamma - \sin\left(\frac{\theta_\sigma - \theta'_\sigma}{2}\right) \sin \beta$$

$$\Sigma_2 = \sin\left(\frac{\theta_\sigma - \theta'_\sigma}{2}\right) \cos \beta \cos \gamma + \sin 2\Omega \cos\left(\frac{\theta_\sigma - \theta'_\sigma}{2}\right) \sin \beta \sin \gamma$$

$$\Upsilon_1 = \cos\left(\frac{\theta_\sigma - \theta'_\sigma}{2}\right) \cos \beta \cos \gamma + \sin 2\Omega \sin\left(\frac{\theta_\sigma + \theta'_\sigma}{2}\right) \sin \beta \sin \gamma$$

$$\Upsilon_2 = \cos\left(\frac{\theta_\sigma - \theta'_\sigma}{2}\right) \sin \beta + \cos 2\Omega \sin\left(\frac{\theta_\sigma - \theta'_\sigma}{2}\right) \sin \beta \sin \gamma.$$

The obtained coefficients recover the results of Ref.s 9, 15 and 24 for N|S, F|S and N|I|S graphene-based configurations, respectively. This can be justified by letting $h \rightarrow 0$ and $\Omega \rightarrow 0$ in the above coefficients for F|I|S graphene-based junctions.

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